Name:____________________________ Date:__________________________

Pre Calculus 12: Section 1.4 Expansions and Compressions of Functions

1. When a function is expanded vertically, what happens to the X and Y coordinates? Explain
   When a function is expanded vertically, all the "y" coordinates will be multiplied by a constant that is representative of the factor of the expansion. Ie: if a function is expanded vertically by a factor of 3, this means that all the "Y" coordinates will be multiplied by 3. Nothing happens to the "x" coordinates.

2. When a function is compressed vertically, what happens to the X and Y coordinates? Explain
   When a function is compressed vertically, then you multiply all the "Y" coordinates by the factor that you compressed by. Ie: a function is compressed vertically by a factor of ½, then multiply all the y coordinates by ½.

3. When a function is expanded horizontally, what happens to the X and Y coordinates? Explain
   Likewise, if we are expanding a function horizontally, multiply all the "x" coordinates by the factor of the expansion. Leave the Y coordinates alone, they don't change.

4. When a function is compressed horizontally, what happens to the X and Y coordinates? Explain

5. Given the transformation $y = f\left(x\right) \to y = f\left(kx\right)$, for what values of "k" will the function be expanded horizontally?
   Use CEEC on the number line. To expand, "K" must be between -1 and 1. Note, if "k" is negative, it will also reflect over the "y" axis [horizontal reflection]

6. Given the transformation $y = f\left(x\right) \to y = f\left(kx\right)$, for what values of "k" will the function be compressed horizontally?
   For the function to compress horizontally, "k" must be either bigger than 1 or less than -1. Again, if "k" is negative, it will have a horizontal reflection over the Y-axis

7. Given the transformation $y = f\left(x\right) \to k \times y = f\left(x\right)$, for what values of "k" will the function be expanded vertically?
   The same rule applies to vertical expansions and compressions. Use CEEC on the number line. If "K" is between 1 and -1, then the function will expand vertically.

8. Given the transformation $y = f\left(x\right) \to y = \frac{1}{k} \times f\left(x\right)$, for what values of "k" will the function be compressed vertically?
   The value of "k" in this question is the same as the previous one. If "k" is on the right side, notice that it is in the denominator!!! So we apply the same rule, Use CEEC. If we want to compress vertically, "K" is either bigger than 1 or less than -1. Again, if "k" is negative, we will have a vertical reflection over the "x' axis.

9. Indicate the transformations from the function on the left to the function on the right. Indicate whether if it is a horizontal/vertical and compression/expansion. Also indicate how the "x" and "y" coordinates will be affected.

a) $y = |x| \rightarrow y = 3|2x|$

List the transformation out: x → 2x (k=2) Horizontal Compression by a factor of ½. All the "x" coordinates will be multiplied by ½

Rewrite this equation to better recognize the vertical transformation: $y = |x| \rightarrow \frac{1}{3}y = |2x|$

y → 1/3 (y) (k=1/3) Vertical expansion by a factor of 3. All the "y" coordinates will be multiplied by 3.

[NOTE: the factor that you expand or compress by is always going to be 1/k. Ask me in class if you don't understand what this means!!!!]

b) $y = \sqrt{x} \rightarrow y = \sqrt{4x}$

Transformation: x → 4x (k=4) Horizontal compression by a factor of ¼ . All the "x" coordinates will be multiplied by ¼

c) $y = \frac{1}{2x-3} \rightarrow y = \frac{3}{4x-3}$

Transformation: x → 2x (k=2) Horizontal compression by a factor of ½. All "x" coordinates will be multiplied by ½

Y → 1/3 y (k=1/3) Vertical expansion by a factor of 3. All y-coordiantes will be multiplied by 3

d) $y = (x-3)^2 \rightarrow y = \frac{1}{5}(2x-3)^2$

Transformation: x → 2x (k=2) Horizontal compression by a factor of ½. All "x" coordinates will be multiplied by ½

Y → 5 y (k=5) Vertical compression by a factor of 1/5. All the "y" coordinates will be multiplied by 1/5.

e) $y = x^3 + x^2 - 5 \rightarrow \frac{1}{3}y = (2x)^3 + (2x)^2 - 5$

Transformation: x → 2x (k=2) Horizontal compression by a factor of ½. All "x" coordinates will be multiplied by ½

y → 1/3y (k=1/3) Vertical expansion by a factor of 3. All "y" coordinates will be multiplied by 3.

f) f) $y = 2^{x+4} \rightarrow y = 4\left(2^{x+4}\right)^3$

For this one, you may need to manipulate the equation first. Here are some exponential rules you should have learnt in grade 10: $\left(a^{b+c}\right)^k = a^{bk+ck}$ and $a^{bk+ck} = a^{bk} \times a^{ck}$

$$y=2^{x+4}\to y=4\left(2^{x+4}\right)^3=4\left(2^{3x+12}\right)=4\left(2^{3x+4+8}\right)=4\left(2^{3x+4}\right)\left(2^8\right)$$

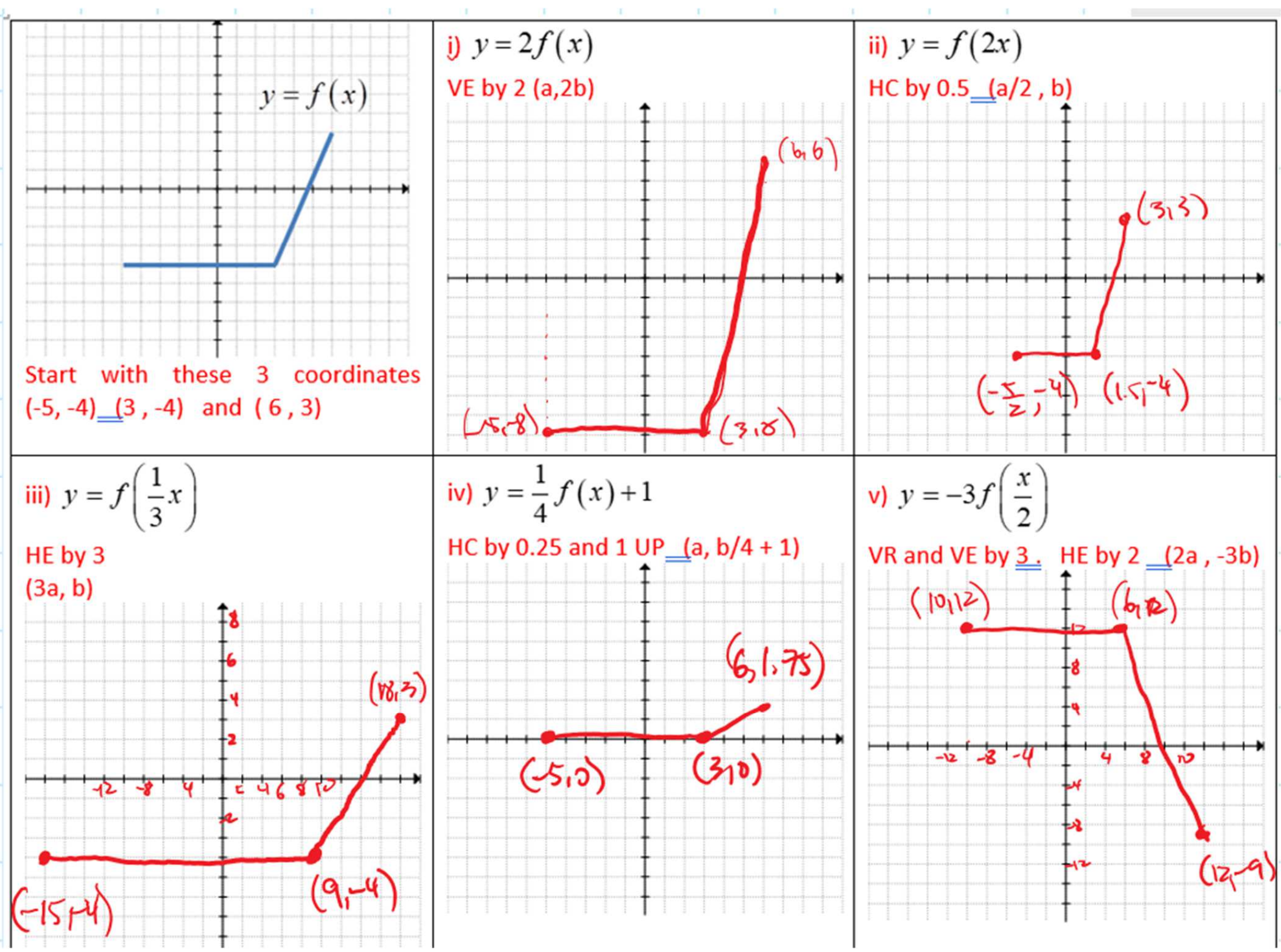
$$y=2^{x+4}\to y=4\left(2^{3x+4}\right)\left(2^8\right)=1024\left(2^{3x+4}\right)$$

$$y=2^{x+4}\to y=1024\left(2^{3x+4}\right)$$

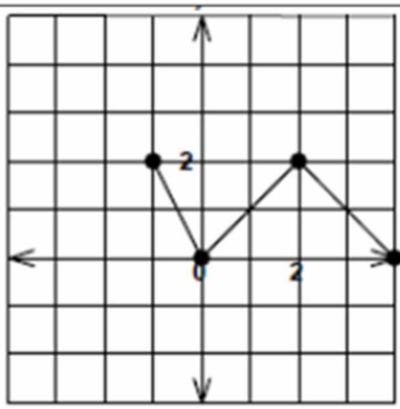
Transformation: x → 3x (k=3) horizontal compression by afactor of 1/3. All the "x" coordinates will be multiplied by 1/3.
y → 1/1024 (y) (k=1/1024) Vertical expansion by a factor of 1024. All the Y-coordinates will be multiplied by 1024.

Note: if you only compress

10. Given the graph of $y=f\left(x\right)$, draw the graph of the foll



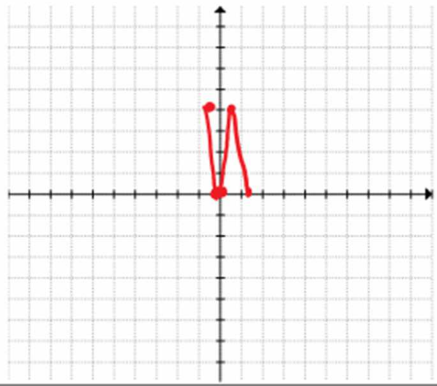
11. Given the graph of $y=f\left(x\right)$, draw the graph of the following:



Start with these four points. Make sure that they are connected in order:
(-1, 2) (0,0) (2,2) and (4, 0)

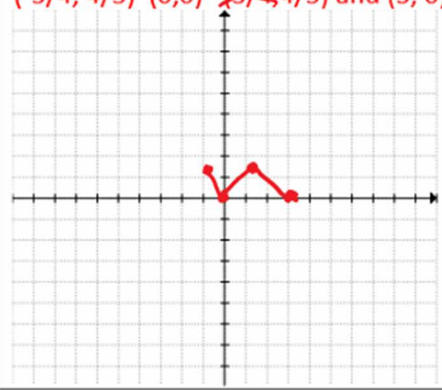
i) $y = 2f(3x)$

VE by 2 and HC by 1/3
(a/3 , 2b)
(-1/3 , 4) (0,0) (2/3,4) and (4/3, 0)



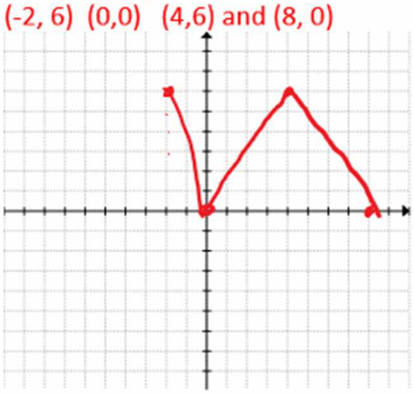
ii) $y = \frac{2}{3} f\left(\frac{4x}{3}\right)$

VC by 2/3 and HC by ¾
(3/4 a , 2/3 b)
(-3/4, 4/3) (0,0) ~~3/4~~ 1.5,4/3) and (3, 0)



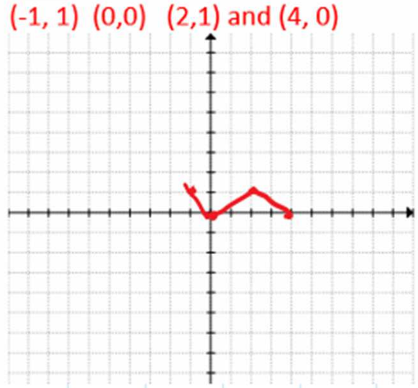
iii) $y = 3f\left(\frac{x}{2}\right)$

VE by 3 and HE by 2
(-2, 6) (0,0) (4,6) and (8, 0)



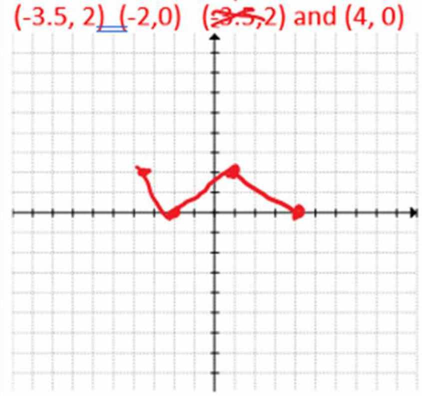
iv) $2y = 3f(x) - 4$

Isolate the y -variable first
$y = 1.5f(x) - 2$
VE by 3/2 and VS 2 Down
(a, 3/2 b – 2)
(-1, 1) (0,0) (2,1) and (4, 0)



v) $y = f\left(\frac{2}{3}(x-2)\right)$

HE by 3/2 and HS 2 LEfT
(3/2 a – 2, b)
(-3.5, 2) (-2,0) (~~-3.5~~ 1,2) and (4, 0)



12. If (a,b) is a point on the graph of $y = f(x)$, determine what this coordinate will become in each of the following functions below:

a) $y = \frac{2}{5} f(3x)$ This is a vertical compression by a factor of 2/5. There is also a horizontal compression by 1/3 $\left(\frac{a}{3}, \frac{2b}{5}\right)$	b) $y = \frac{1}{2} f\left(\frac{4}{5}x\right)$ Vertical compression by ½ HOriztonal expansion by 5/4 $\left(\frac{5a}{4}, \frac{b}{2}\right)$

c) $y=-0.75f(2x)$ Vertical reflection and vertical compression by ¾ Horizontal compression by ½ $\left(\frac{a}{2},\frac{-3b}{4}\right)$	d) $y=-1.25f(x-3)-2$ Vertical expansion by 5/4 and vertical reflection Horizontal shift of 3 right Vertical shift of 2 down $\left(a+3,-\frac{5b}{4}-2\right)$

13. Given $y=f(x)$, indicate what the new equation will be after the transformations given in the order stated:

a) $f(x)=2x+3$ 1. x → x/3 $y=2\left(\frac{x}{3}\right)+3$ 2. y → y-5 $y-5=\frac{2x}{3}+3$ $y=\frac{2x}{3}+8$	1. A horizontal expansion by a factor of 3 x → x/3 2. Then shifted up by 5 units y → y-5
b) $f(x)=(x-3)^2-4$ 1) y → 2y and y → -y $-2y=(x-3)^2-4$ 2) y → y + 6 $-2(y+6)=(x-3)^2-4$ 3) Simplify: $(y+6)=-\frac{1}{2}(x-3)^2+2$ $y=\frac{-1}{2}(x-3)^2-8$	1. A vertical compression by a factor of 0.5 y → 2y 2. A vertical reflection over the “X” axis y → -y 3. Shift of 6 units down y → y + 6
c) $f(x)=\sqrt{x+2}+4$ 1. HR x → -x $y=\sqrt{-x+2}+4$ 2. x → 3x $y=\sqrt{-3x+2}+4$ 3. x → x + 3 $y=\sqrt{-3(x+3)+2}+4$ 4. Simplify: $y=\sqrt{-3x-9+2}+4$ $y=\sqrt{-3x-7}+4$	1. A Reflection in the y-axis and HR x → -x 2. A Horizontal compression by a factor of 1/3. x → 3x 3. A shifted 3 units left. x → x + 3
d) $f(x)=2^x+3$	1. A reflection in both the “x” and “y” axis

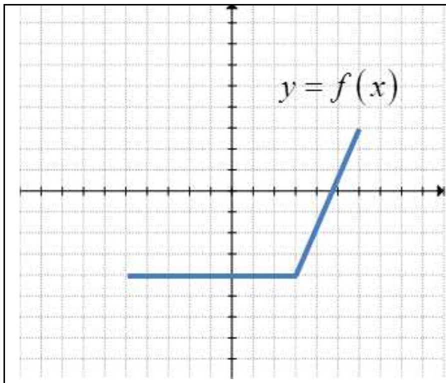
1. X → -x y → -y $-y=2^{-x}+3$ 2. x → 0.5x $-y=2^{-0.5x}+3$ 3. y → y + 11 $-(y+11)=2^{-0.5x}+3$ 4. Simplify: $(y+11)=-2^{-0.5x}-3$ $y=-2^{-0.5x}-14$	X → -x y → -y 2. A horizontal expansion by a factor of 2, x → 0.5x 3. A shifted of 11 units down y → y + 11
e) $x^2+(y-1)^2=9$	1. A reflection in the "y" axis, 2. A Horizontal expansion by a factor of 2 and 3. A vertical compression by a factor of 0.5.

14. Given the equation $f(x)=\sqrt{3x-2}$, if we were to perform two transformations:

A) A horizontal compression by a factor of 0.5 B) A horizontal shift of 4 units right

What would the function look like if we performed "A" first and then "B"?

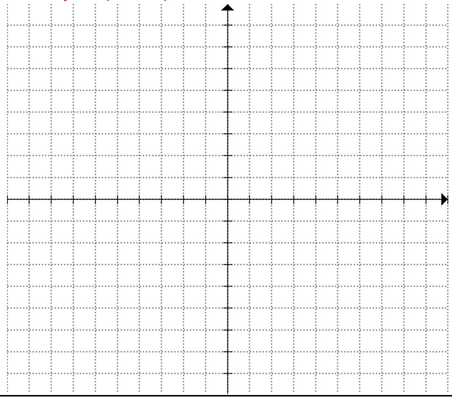
B) What would the function look like if we performed "B" first and then "A"?



Start with these 3 coordinates (-5, -4) (3 , -4) and (6 , 3)

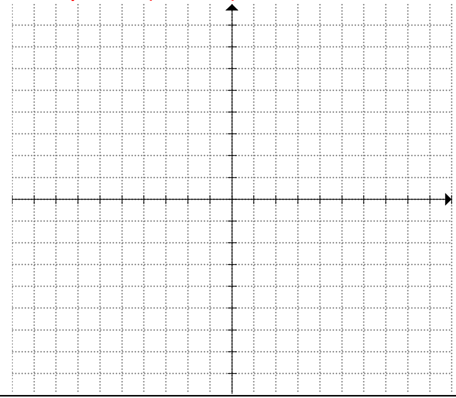
i) $y = 2f(x)$

VE by 2 (a,2b)



ii) $y = f(2x)$

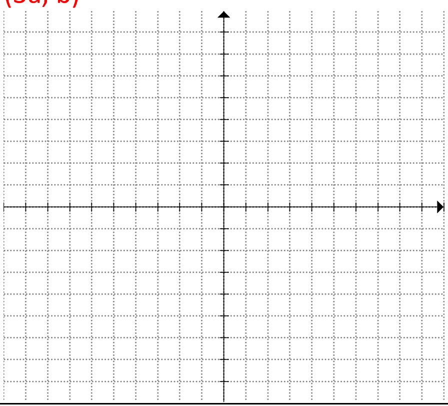
HC by 0.5 (a/2 , b)



iii) $y = f\left(\frac{1}{3}x\right)$

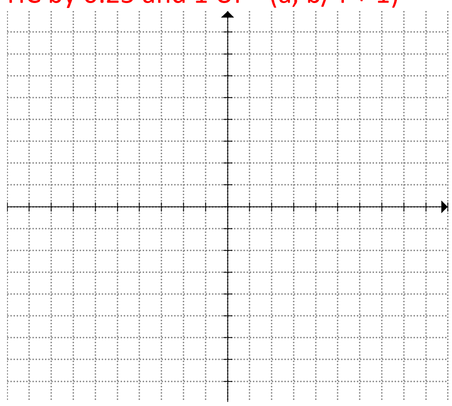
HE by 3

(3a, b)



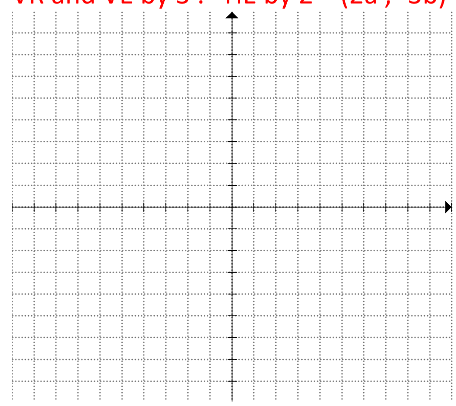
iv) $y = \frac{1}{4}f(x) + 1$

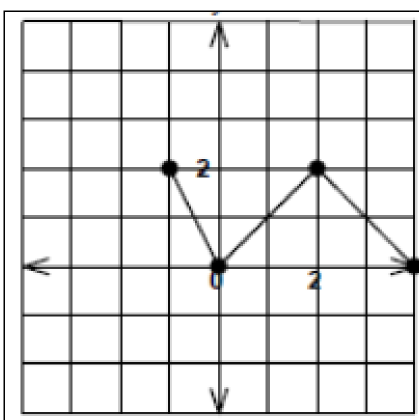
HC by 0.25 and 1 UP (a, b/4 + 1)



v) $y = -3f\left(\frac{x}{2}\right)$

VR and VE by 3 . HE by 2 (2a , -3b)





Start with these four points. Make sure that they are connected in order:

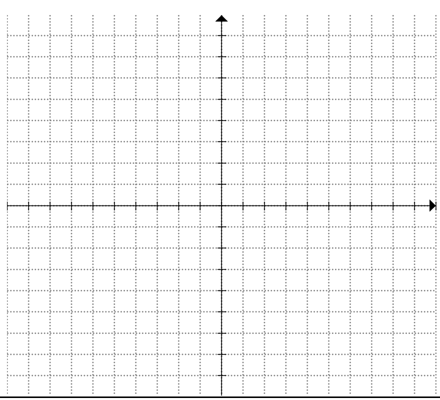
(-1, 2) (0,0) (2,2) and (4, 0)

i) $y = 2f(3x)$

VE by 2 and HC by 1/3

(a/3 , 2b)

(-1/3 , 4) (0,0) (2/3,4) and (4/3, 0)

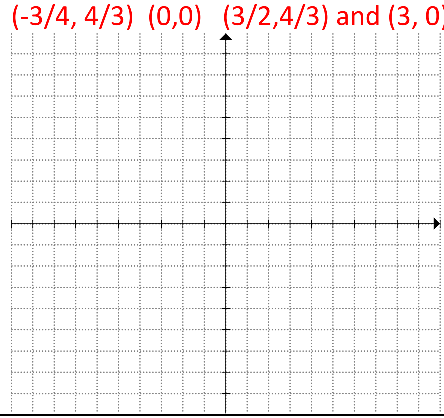


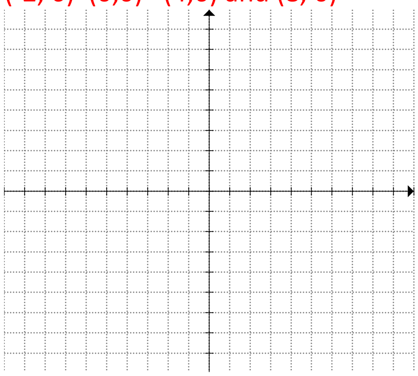
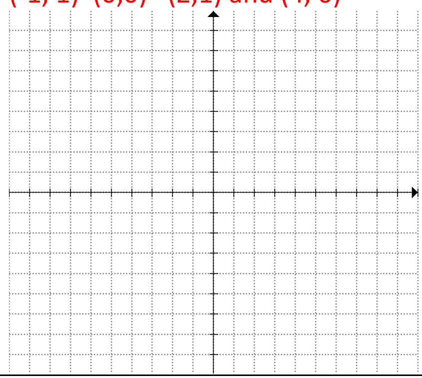
ii) $y = \frac{2}{3}f\left(\frac{4x}{3}\right)$

VC by 2/3 and HC by ¾

(3/4 a , 2/3 b)

(-3/4, 4/3) (0,0) (3/2,4/3) and (3, 0)



iii) $y = 3f\left(\frac{x}{2}\right)$ VE by 3 and HE by 2 (-2, 6) (0,0) (4,6) and (8, 0) 	iv) $2y = 3f(x) - 4$ Isolate the y -variable first $y = 1.5f(x) - 2$ VE by 3/2 and VS 2 Down (a, 3/2 b – 2) (-1, 1) (0,0) (2,1) and (4, 0) 	v) $y = f\left(\frac{2}{3}(x-2)\right)$ HE by 3/2 and HS 2 LEfT (3/2 a – 2, b) (-3.5, 2) (-2,0) (1,2) and (4, 0)